PSI AP Physics 1
Rotational Motion AP Style Free Response
Name_____

1. Students wish to determine the moment of inertia of a pulley. They cannot measure the mass of the pulley because it is attached to a stand and cannot be removed.

    a. Put a check next to each to the items that are needed for this investigation.

    ___ Stopwatch

    ___ Meter stick

    ___ String

    ___ Hanging masses

    ___ Carts

    ___ Track

    ___ Motion sensor

    b. How would this equipment be used? What quantities would be measured?

    c. Describe an overall procedure for obtaining the necessary data to determine the moment of inertia of the pulley. Include a labeled diagram of your set up and enough detail so that someone else could carry out the procedure.

    d. When the students carry out their experiment, they realize that there is significant friction between the pulley and string. How will this source of error affect their answer?

2. Students in a lab are investigating the moments of inertia of various objects: a solid cylinder, a hollow cylinder, and a solid sphere. (All objects have the same mass.) They are given an inclined plane and have access to materials that are typically found in a high school lab. They need to find and compare the moments of inertia of the three objects.

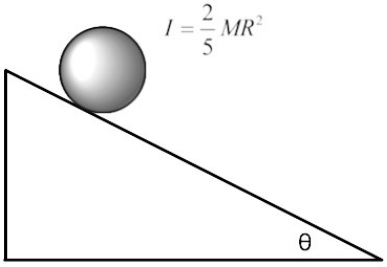
   a. Describe your experimental setup and procedure, including any additional pieces of equipment you would need and the kind of data you would record. Include enough detail that another student could follow and complete the experiment successfully.

   b. Rank the moments of inertia for the three objects.

      ___ Solid cylinder ___ Hollow cylinder ___ Solid sphere

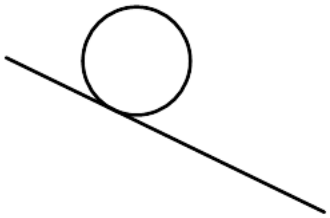
      Briefly explain your reasoning without deriving or using the exact expressions for the moments of inertia of the three objects.

   c. Which of the three objects would have the fastest speed at the bottom of the incline? Briefly explain your reasoning in terms of energy.



3. A sphere, with a mass *M* and a radius *R*, rolls down a ramp that makes an angle θ with the horizontal. The moment of inertia of the sphere is $\frac{2}{5}MR^2$. The sphere rolls without slipping.

   a. On the diagram below, draw and label all the forces (not components) acting on the wheel as it rolls. Clearly indicate at which point on the sphere the force is acting.



   b. As the sphere rolls down the ramp, which force causes a change in its angular velocity with respect to its center of mass? Briefly explain your reasoning.

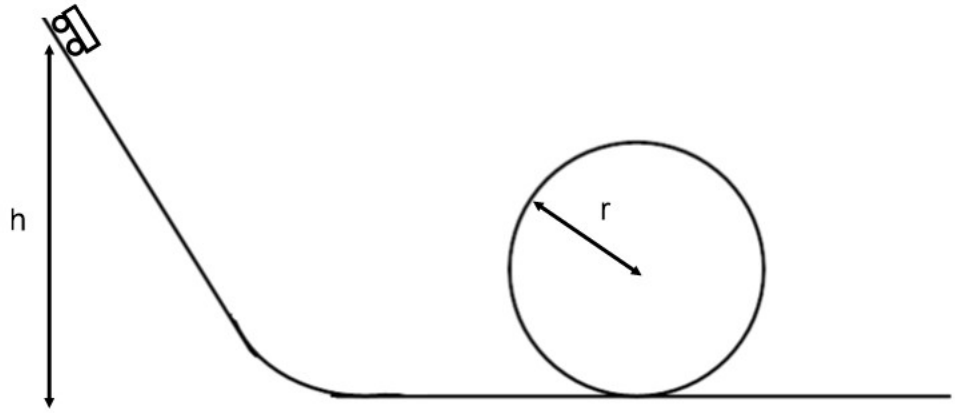
   c. In terms of *M, R, θ*, and fundamental constants, determine the minimum force of friction necessary to keep the sphere rolling without slipping.

   d. In terms of *M, R, θ*, and fundamental constants, determine the acceleration of the sphere.

   e. The sphere is now replaced with a cylinder of the same mass and radius. Will the cylinder have an acceleration greater than, less than, or equal to the acceleration of the sphere?

      _____ Greater than _____ Less than _____ The same

Briefly explain your reasoning.



4. A toy car is at the top of a hill of height, h, on a track with a loop of radius, r, at the bottom as show in the diagram. The car is released from rest and makes it around the loop and returns to the bottom. Assume the masses of the cart's wheels are negligible, as are any frictional or drag forces.

   a. Indicate whether the net force on the car at the top of the loop will be greater than, less than, or equal to the net force on the car at the bottom of the loop.

   ___ Greater that at the bottom

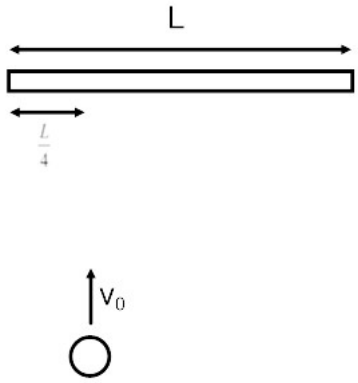
   ___ Less than at the bottom

   ___ The same as at the bottom

   Explain how you arrived at your answer.

   b. In terms of r, determine the minimum value of h necessary for the car to make it around the loop, returning to the bottom, without falling off.

   c. If the masses of the wheels of the cart were not negligible, how would it effect your answer to part (b). Briefly explain your reasoning.



5. A rod of length L and rotational inertia I rests on a frictionless surface. A disk of mass m slides towards the rod and will hit the rod at a point that is L/4 from one end. Before this happens, two students discuss what will happen after the disk hits the rod.

   Student 1 says, since the rod is not attached to the surface at any pivot point, it will not rotate but simply move because linear momentum is conserved.

   Student 2 says, since the disk is hitting the rod off center, it will only rotate and not move because angular momentum is conserved.

   a. Indicate any aspects of student 1's claim that are correct.

   b. Indicate any aspects of student 1's claim that are incorrect. Explain your reasoning.

   c. Indicate any aspects of student 2's claim that are correct.

   d. Indicate any aspects of student 2's claim that are incorrect. Explain your reasoning.

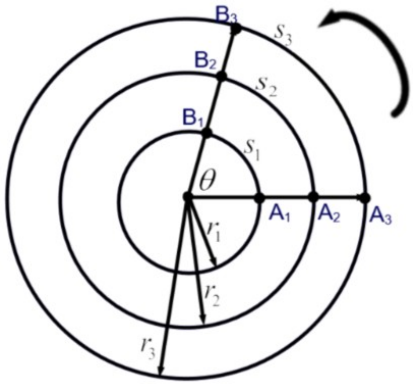
   e. For the next experiment, one of the students attaches a pivot to the right end of the rod so that it can rotate but not move. Indicate which outcome will result in a greater angular velocity for the rod.

      ___ The disk bounces back.

      ___ The disk sticks to the rod.

Briefly explain your reasoning without deriving an equation for the angular velocity of the rod.

6. A wheel, labeled with points 1, 2, and 3, each starting at angular position A, is rotated ending at angular position B. In this wheel, the distances between points 1, 2, and 3 are the same.



a. Graph the linear velocity vs angular velocity for points 1, 2, and 3 on the same graph. Which plot has the largest slope? The smallest slope? What value does the slope indicated?

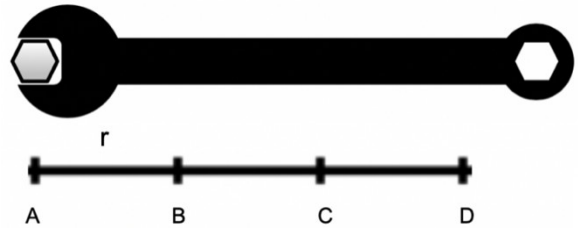
b. A student claims that the centripetal force on point 3 is triple the centripetal force on point 1 because point 3 is three times as far from the axis of rotation as point 1, making the linear velocity $\sqrt{3}$ times more.

   i. Which aspects of the students reasoning, if any, are correct? Explain using only words and no equations.

   ii. Which aspects of the students reasoning, if any, are correct? Explain using only words and no equations.

c. To make the wheel turn, it must be pulled by a rope that is tied either to point 1, 2, or 3. From which point is it easiest to turn the wheel? Explain your answer using only words and no equations.

7. A wrench is used to loosen a bolt. The wrench can be pulled straight downwards with a force F at points A, B, C, or D.



a. At which point is it easiest to turn the bolt? Explain your reasoning using words, not equations.

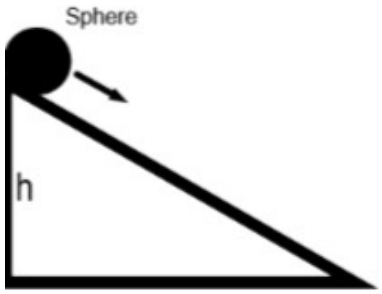
b. The instant that the force is applied straight down, what type of acceleration is also straight down? Select one choice and explain your reasoning.

____centripetal acceleration ____tangential acceleration ____linear acceleration

c. For A, B, C, and D, construct a graph of applied torque and angular acceleration. Which plot has the largest slope? The smallest slope? What value does the slope indicate?

d. The wrench is replaced with a longer wrench, and is used to loosen the same bolt. Will this wrench require more or less force to turn the bolt? Explain.

8. Using the experimental set up shown, two students want to find the moment of inertia for a sphere.



a.

i. State one or more physics laws or principles that can be used to determine the moment of inertia.

ii. Using your statements from (a)(i), derive an equation, or a set of equations, that the students can use with the data they collect to find the moment of inertia of the sphere.

b. Design an experiment that can be used to determine the moment of inertia for the sphere. Assume equipment usually found in a physics laboratory is available.

i. In the table below, indicate what quantities need to be measured and the equipment required to measure those quantities. You do not need to use every row, and you can add more rows if needed.

Quantity to be Measured	Equipment for Measurement

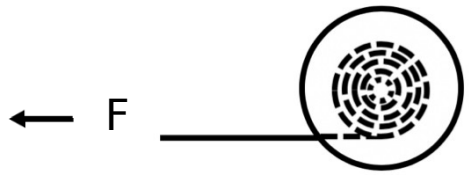
ii. Describe the procedure to be used to determine the moment of inertia of the sphere. In your description, include any experiment uncertainties and how those uncertainties can be reduced.

c. Now a hoop is used in the experiment instead of a sphere. How do you expect the results to compare? Explain.

9. A student places a yoyo on a rough desk. He pulls on the yoyo's string, which rolls without slipping, from various positions.

a.

i. He pulls on the string to the left, as shown. Which direction do you expect the yoyo to move? Explain your reasoning.



ii. Draw a simpllified diagram that shows the torques caused by all the forces acting on the yoyo and causing it to move.

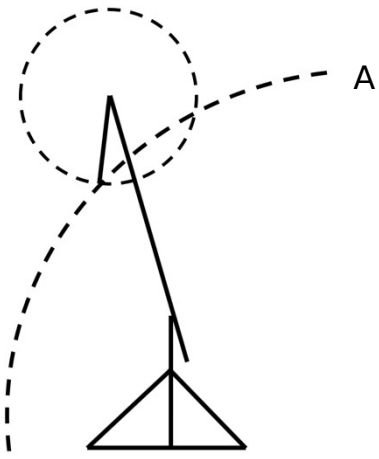
b.

i. Now the student pulls upward on the string, as shown. Which direction do you expect the yoyo to move? Explain your reasoning.

ii. Draw a simpllified diagram that shows the torques caused by all the forces acting on the yoyo and causing it to move.

c. If the string were attached on the outer edge of the yoyo instead of in the center, would the student get the same results as she did for parts (a) and (b)? Explain.

10. A student wants to design a trebuchet, and she draws the sketch below. A basket containing a ball is attached at the end of the small arm. The larger dotted arc is the circular motion of the large arm with radius 5r, and the small dotted circle is the circular motion of the small arm with radius r. The large arm is pulled up in the clockwise direction and stops at point A. At that point the basket continues rotating as the small arm pivots on the large arm. At the top of the circle, the ball flies out of the basket.



a. Without using equations, compare the angular velocity of the large arm to the small arm while the large arm was moving.

b. Derive an equation to find the angular velocity of the small arm after the large arm stops moving.

c. Is the angular momentum conserved between the small arm and the large arm? Explain.

d. Is it possible to determine the moment of inertia for the large arm? Explain. How would it compare to the moment of inertia of the smaller arm?

Answers:

1.
   a. Stopwatch, meter stick, hanging masses, string, pulley
   b. Pulley and string – to create a pulley-mass system
      Hanging masses – to attach on either side of the string in the pulley-mass system (m_1, m_2)
      Meter stick – to measure the distance the heavier mass travels until it hits the ground (x), and the radius, R, of the pulley
      Stopwatch – to measure the time it takes the heavier mass to reach the ground (t)
   c. Attach two hanging masses of known mass on opposite ends of the string, place the string over the pulley and hold the heavier mass, m_2, at a distance x above the ground. Measure x using the meter stick. Release the system and record the time, t, it takes for m_2 to reach the ground. Find the linear acceleration using $x=\frac{1}{2}at^2$. Find the tension in each string by substituting in the calculated acceleration, a, into the following Newton's Second Law equations for each mass:

      $F_{T1} - m_1g = m_1a$
      $m_2g - F_{T2} = m_2a$

      Find the moment of inertia of the pulley using:

      $$\sum \tau = I\alpha$$
      $$(F_{T2} - F_{T1})R = I\frac{a}{R}$$
      $$I = \frac{R^2}{a}(F_{T2} - F_{T1})$$

d. The acceleration of the masses was measured and calculated using the meter stick, timer and Newton's Second Law. The moment of inertia was then calculated assuming that the only forces acting on the pulley were the tension forces. Since friction is present in the string and pulley interface, the pulley would require less torque to rotate it without this friction. Thus, the calculated moment of inertia is higher than the true moment of inertia of the pulley.

2.

a. Place the solid cylinder at the top of an incline. Using a meter stick, measure the height of the ramp and label the height, *h*. Place a motion sensor a few centimeters from the bottom of the incline. Release the cylinder so that it rotates. Measure its translational velocity, *v*, at the bottom of the incline using the motion sensor. Repeat with the hollow cylinder and the sphere. Calculate the moment of inertia using the conservation of energy

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

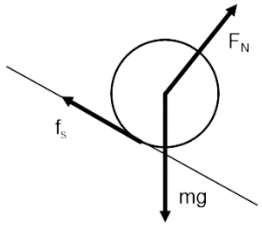
b. 2 – solid cylinder, 1 – hollow cylinder, 3 – sphere

Moment of inertia is defined as the measure of an object's resistance to angular acceleration due to a net applied torque. The easier it is to rotate an object (creating an angular acceleration), the lower its moment of inertia. Of the three objects, the sphere is the easiest to rotate so it will have the lowest moment of inertia. An object's moment of inertia depends on its mass distribution. The more mass of an object that is closer to the axis of rotation results in a lower moment of inertia (it's easier to rotate). The solid cylinder will have a lower moment of inertia than the hollow sphere because its mass is distributed closer to the center of rotation.

c. The sphere would travel the fastest at the bottom because it has the lowest moment of inertia. Less of the initial energy will be required to rotate the sphere, resulting in a greater translational kinetic energy and a greater velocity at the bottom of the incline.

3.

a.



b. The force of friction causes the change in angular acceleration. Because F_N and mg pass through the center of mass, the friction force is the only force that can create a torque about the center of mass.

c. When an object rolls without slipping, the friction between the object and the surface is static friction as the velocity at the interface point is zero. When the maximum static friction force is not great enough to maintain the zero velocity at the interface, then the object slips. At the point where it starts slipping, $a = R\alpha$ is no longer valid. The minimum force to start the slipping is where that condition just barely holds. Calculate using Newton's Second Law in linear and rotational forms.

$$\sum F = ma \qquad \sum \tau = I\alpha$$

$$Mg\sin\theta - f_s = Ma \qquad f_s R = \frac{2}{5}MR^2\frac{a}{R}$$

$$f_s = \frac{2}{5}Ma$$

$$Mg\sin\theta - \frac{2}{5}Ma = Ma$$

$$a = \frac{5}{7}g\sin\theta$$

$$f_s = \frac{2}{5}Ma = \frac{2}{5}M\left(\frac{5}{7}g\sin\theta\right) = \frac{2}{7}Mg\sin\theta$$

d. $\sum F = ma \qquad \sum \tau = I\alpha$

$$Mg\sin\theta - f_s = Ma \qquad f_s R = \frac{2}{5}MR^2\frac{a}{R}$$

$$f_s = \frac{2}{5}Ma$$

$$Mg\sin\theta - \frac{2}{5}Ma = Ma$$

$$a=\frac{5}{7}g\sin\theta$$

e. Less than the sphere. The cylinder has a moment of inertia, *I*, greater than that of the sphere. Since *I* is inversely proportional to *α*, the cylinder will have a smaller α. The object is not slipping so *α = a/R*, thus the cylinder will have a lower acceleration.

4.

a. The net force on the top of the loop will be less than at the bottom of the loop.

The net force on the car at any point will be $\frac{m v^2}{R}$, as the car is in centripetal motion. Using conservation of energy, at the bottom of the loop, all the potential energy will be turned into kinetic energy, so the velocity will be at a maximum. In contrast, at the top of the loop, there is some potential energy, so the kinetic energy (and the velocity) will be lower.

b. Min: when $F_N = 0$

$$mg=ma$$

$$g=\frac{v^2}{R}$$

$$v^2=gR$$

Conservation of energy:

$$mgh=\frac{1}{2}m v^2+mg(2R)$$

$$h=\frac{5}{2}R$$

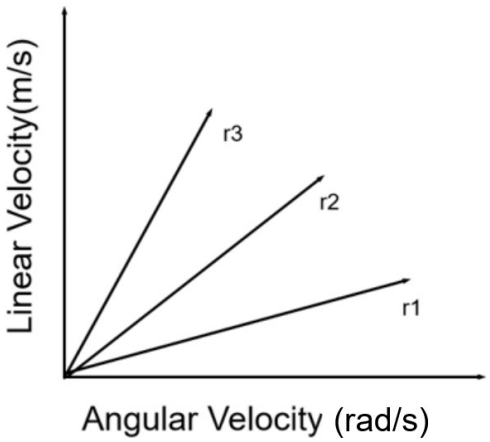
c. If the mass of the wheels was not negligible, then at the top of the loop, the car would also have rotational kinetic energy due to the rotational moment of inertia of the wheels. The height of the hill would need to be higher to make up for the energy used to rotate the wheels.

5.

a. Student 1's claim that linear momentum is conserved is correct.
b. Student 1's claim that the rod does not rotate is incorrect. The disk has an angular momentum about the rod when it is approaching the rod. Because no external torque is acting on the rod-disk system, angular momentum must also be considered, and the rod will rotate after the disc collides with it.
c. Student 2's claim that the angular momentum is conserved is correct.
d. Student 2's claim that the rod will only rotate is incorrect. No outside forces are acting on the system, so linear momentum must also be conserved. Thus, the rod will also move after the collision.

e. The disk bounces back. When the disk bounces back, it bounces with a negative angular momentum. For angular momentum to be conserved, the momentum of the rod must be greater than if the disk stuck to the rod, resulting in a greater angular velocity.

6.



a. $\omega=\frac{v}{r}, r=\frac{v}{\omega}$; the slope of a linear vs. angular velocity graph is the radius. The largest slope is the largest radius or r_3. The smallest slope is the smallest radius r_1.

b.

i. The student is correct that the centripetal force is 3 times greater at radius r_3 than at r_1. The centripetal force is equal to $mr\omega^2$, so when the radius is 3 times as great, the force is also 3 times as great.

ii. The student is incorrect that the velocity is $\sqrt{3}$ times larger. Since rotational velocity is the same, then the velocity would have to be 3 times as greater, since the radius is 3 times as great at point 3.

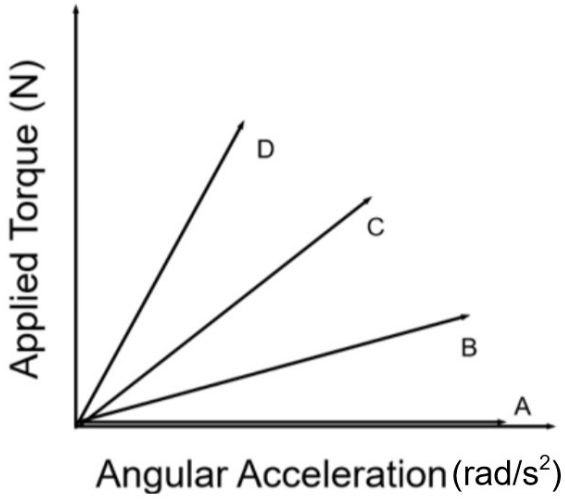
c. The wheel would be easiest to turn if the rope were to be fastened to point 3 which has the longest radius. This is because torque is proportional to the length of the radius from the axis of rotation. With more torque due to its longer radius, pulling from the point 3 is easier than pulling from a point with a smaller radius and less torque.

7.

a. It is easiest to turn the bolt at point D. As the force is applied farther away from the axis of rotation, the torque increases. The torque of the turning bolt is proportional to the radius of the wrench. As the

radius increases, there is more torque for the same amount of force applied, making it easier to turn the wrench at the end.

b. The force applied by the wrench is straight down, so it is the tangential force. Centripetal force is always towards the center, so that is not correct. The linear acceleration represents the vector sum of the tangential force and the centripetal force so that is not correct – it is pointed inwards, and forward of the centripetal force.



c. The largest slope will be at point D as it has the greatest net torque. The smallest slope is at point A which has the smallest net torque (torque is proportional to the radius from the axis of rotation. The slope of this graph will indicate the moment of inertia. $\Sigma\tau = I\alpha$, $I = \frac{\Sigma\tau}{\alpha}$

d. This wrench will require less force to turn the bolt because it has a longer handle. The torque is proportional to the radius of the wrench. Therefore, it will require less force to achieve the same level of torque on a longer wrench.

8.

a.

i. The conservation of energy can be used to determine the moment of inertia.

ii. $U_G = KE_T + KE_R$, $mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$, $\frac{1}{2} I \omega^2 = mgh - \frac{1}{2} m v^2$,

$I = \frac{2m(gh - \frac{1}{2} v^2)}{\omega^2}$, $\omega = \frac{v}{r}$, $I = m r^2 (\frac{2gh}{v^2} - 1)$

b.

i.

Quantity to be Measured	Equipment for Measurement
Mass of sphere	Scale
Radius of sphere	Meterstick/ruler
Height of ramp	Meterstick/ruler
Velocity at bottom of incline	Motion Sensor

ii.

1) Use the scale to measure the mass of the object (sphere).
2) Use the meterstick/ruler to measure the radius of the sphere and the height of the inclined plane.
3) Place the motion sensor at bottom of ramp.
4) Place the object at the top of the ramp and release from rest. Use the motion sensor to measure the velocity at the bottom.
5) Use the conservation of energy where all the gravitational potential energy is transformed into translational and rotational kinetic energy at the bottom of the inclined plane. Assume that the sphere is rolling without slipping so $v = r\omega$.

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} I \frac{v}{R^2}^2$$

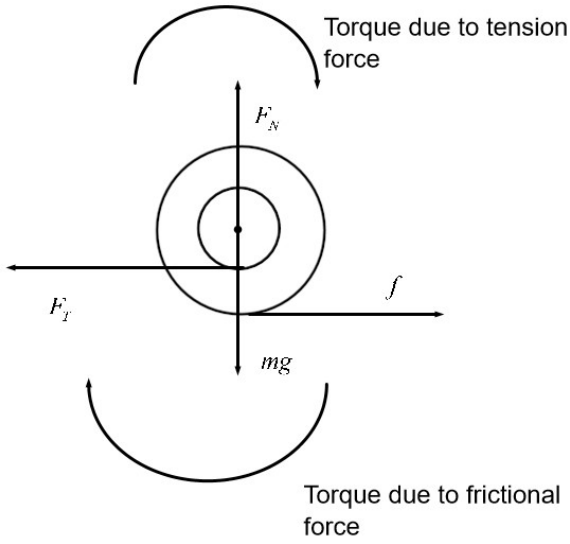
Solve this equation for I and substitute in the measured values.
6) Minimize the uncertainty in the measurement by taking multiple trials and averaging the values of the moment of inertia calculated in each trial.

c. The moment of inertia of the hoop will be greater than the moment of inertia of the sphere because more of the hoop's mass is farther from the axis of rotation.

9.

a.

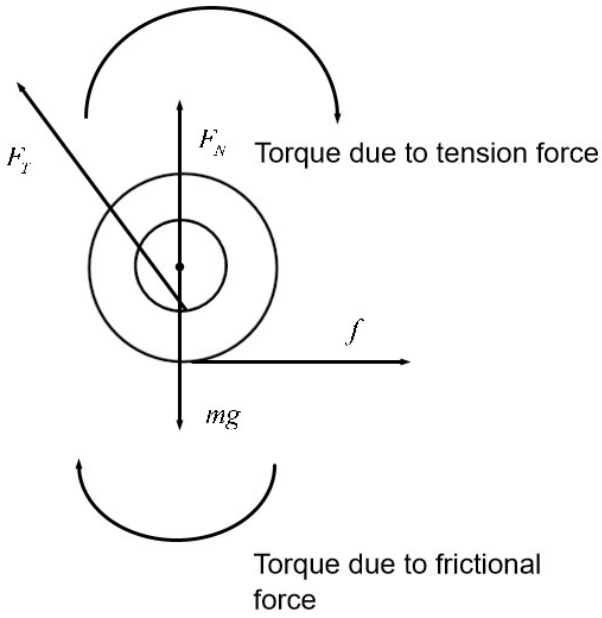
i. When the student pulls the string to the left, this causes the yoyo to begin rotating in the counterclockwise direction, moving to the left. The yoyo rotates without slipping and the torque provided by the friction on the surface is more powerful than the rotation provided by the string from an angle below the center of mass line.



ii.

b.

i. If the student pulls upward on the string, the yoyo would move in the clockwise is direction, moving to the right. This is because the yoyo rotates without slipping and the string is pulled from an angle which is above the center of mass line. This gives it sufficient torque in the clockwise direction to overcome the torque in the counterclockwise direction which is provided by friction.



ii.

c. If strings were attached on the edge instead of in the center, the student might get different results as the rotation of the yoyo is dependent on the center of mass line relative to the string. If the string was placed in a position which was further offset from the center of mass, the torque applied to overcome the torque provided by the frictional force would be greater (torque is proportional to the distance from the axis of rotation.) Therefore, the student could pull the string at a much smaller angle to pull the yoyo in a direction which would allow it to unravel.

10.

a. The angular velocity of the larger arm is smaller than the angular velocity of the small arm because of the definition of angular momentum. Both arms are moving with the same angular momentum, so the arm with the greater moment of inertia (the large arm) will have a smaller angular velocity: $L = I\omega$.
b. When the first arm stops moving, all momentum from the initial movement of the large arm is transferred to the small arm $I_1 \omega_1 = I_2 \omega_2$, and solve for the angular velocity, ω_2, of the small arm: $\omega_2 = \frac{I_1 \omega_1}{I_2}$. $I_1 > I_2$, so the angular velocity of the small arm will be greater than the angular velocity of the large arm before it stops.
c. Angular momentum is conserved as this is a closed system. There are no additional torques and all torque applied to the second arm is due to the rotation of the first arm.

d. The moment of inertia for the longer arm is greater than the moment of inertia for the shorter arm because the moment of inertia is proportional to the square of the radius of rotation. The radius of rotation is dependent on the length of the arm. Therefore, a longer arm has a larger moment of inertia.